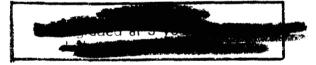
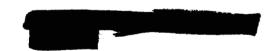


FOR APOLLO/SATURN 204

DECEMBER 15, 1966



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Prepared by Bell Telephone Laboratories, Incorporated on behalf of Bellcomm, Inc.

TABLE OF CONTENTS

			Page
I.	Inti	roduction	1
II.	Refe	erence Trajectory Description	3
III.	Sim	ulation Description	6
	Α.	Missile Dynamics	6
	B.	Inertial Platform Simulation	9
	C.	Accelerometer Processing	11
	D.	Navigation Equations	12
	Ε.	Guidance Equations	12
IV.	Veh	icle Data	14
٧.	Err	or Analysis Results	17
	Α.	Errors at S-IVB Cutoff	17
	В.	Errors at 640 Seconds	19
VI.	Ana	lytical Investigations	32
	Α.	Noise Models	32
	В.	Noise and Dynamic Response of the M/F Filter	34
	С.	Noise Response of the Navigation Equations	38
	D.	The Effect of Noise on the Prediction of the Cutoff Time	42
	Ref	Cerences	
	App	pendix	
	Fig	rures 1-13	

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I. Introduction

This report describes an error analysis of the Apollo/Saturn 204 test mission launch vehicle trajectory. The purpose of this study was to evaluate uncertainties at launch vehicle cutoff due to inertial platform and performance uncertainties. The study was performed by the Control Systems Analysis Department at Bell Telephone Laboratories, Incorporated on behalf of Bellcomm, Inc.

The AS-204 mission is a manned orbital flight designed to test spacecraft operations and launch vehicle systems performance for an orbital mission. The launch vehicle for AS-204, consisting of an S-IB first stage and an S-IVB second stage, will be launched from Launch Complex 34 at Cape Kennedy along a flight azimuth of 72 degrees. In this study, the SA-204 Launch Vehicle Operational Flight Trajectory document was used as a reference for both vehicle data and the nominal trajectory description.

The effects of platform and performance uncertainties are determined by simulating perturbed trajectories for $\pm 3\sigma$ magnitudes of each error source. The resulting 3σ variations in significant trajectory parameters are determined both at S-IVB cutoff and at a fixed time (50 seconds) after nominal S-IVB cutoff. In addition, three analytical

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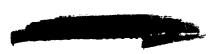
investigations were performed. These include:

- (1) Noise and dynamic response of the M/F filter.
- (2) Noise response of the navigation equations.
- (3) The effect of noise on the prediction of cutoff time.

The noise source considered in these studies is the accelerometer measurement quantization.

Using three-sigma platform and performance uncertainties obtained from references 10 and 11, the following are the three-sigma deviations in the actual state at S-IVB cutoff:

Radius	1492.6 ft
Velocity magnitude	3.981 !ft/sec
Flight path angle	.0131 deg
Inclination	.0068 deg
Node angle	.0252 deg
Time from launch	19.217 sec
S-IVB fuel reserve	1579.5 lbs

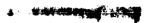


II. Reference Trajectory Description

The nominal closed loop trajectory used for this study is a match of the SA-204 Launch Vehicle Operational Flight Trajectory. To develop this nominal trajectory, an open loop trajectory was first generated which matches the significant parameters on the reference.

The pitch profile of the open loop trajectory consists of a 10 second vertical rise followed by an instantaneous change in pitch (kick angle) to initiate a gravity turn. The gravity turn is flown until 133 seconds at which time the attitude is fixed. At 185.4 seconds, the vehicle is given a second instantaneous pitch change followed by a constant pitch rate until S-IVB cutoff.

The kick angle at 10 seconds was selected to match the reference altitude at 185.4 seconds (321,405 ft). The pitch angle at 185.4 seconds together with the constant pitch rate and S-IVB cutoff time were chosen to satisfy the radius (21,442,123 ft), velocity (25,702.53 ft/sec), and inertial flight path angle (0.0 deg.) at S-IVB cutoff. These constraints, together with a launch azimuth of 72 degrees and no roll or yaw maneuvers, yield an orbital plane with an inclination of 32.469 degrees and an angle from the launch meridian to the descending node of 123.315 degrees.



- 4 -



The resulting open loop reference trajectory was then processed to calculate all required guidance constants. A complete closed loop simulation was then performed, using the 2-stage IGM equations, to generate the nominal trajectory shown in Table 1.

TABLE 1
Closed Loop Reference Trajectory

<u>Event</u>	Time	Latitude	Longitude	Altitude	Velocity	Flight Path Angle	Flight Path Azimuth
	sec	deg	deg	ft	ft/sec	deg	deg
LAUNCH	0	28.36	-80.56	40.	1341.76	0.	90.02
IECØ	141.65	28.52	-79•99	189762.	7586.15	25.91	75.83
siøff	144.60	28.54	-79.94	199591.	7753.06	25.52	75•77
s4øn	149.00	28.56	-79.87	214027.	7693.78	24.67	75.81
MIXI	153.50	28.58	-79•79	228322.	7730.06	23.91	75.8 2
JETISG	157.20	28.60	- 79 · 73	239822.	7771.96	23.30	75.83
JETLES	185.40	28.73	-79.22	320557•	8143.38	18.98	75.88
MIX2	475.00	_30.78	-69.94	578143.	18183.15	-1.23	79.38
s4øff1	589.495	31.79	-62.79	536098.	25702.64	0.	83.01





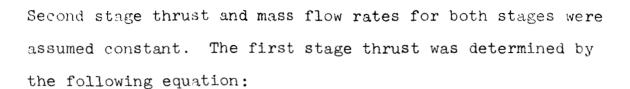
III. Simulation Description

The simulation required for the error analysis was performed using the Bellcomm Apollo Simulation Program (BCMASP).² This program is a three degree of freedom simulator which generates the trajectory by numerical integration of all significant accelerations acting on the vehicle. Since BCMASP is described elsewhere in detail,^{3,4} only those aspects of the program that relate directly to the error analysis will be discussed in this report.

A functional diagram of the calculations performed in each guidance cycle by the simulation program is given in Figure 1. As illustrated in this diagram, the missile dynamics are simulated to obtain the actual position, velocity, and the integral of the nongravity acceleration. The accelerometer measurements are simulated by converting the integral of nongravity acceleration to platform coordinates and adding errors associated with the platform gyros and accelerometers. The accelerometer processing, navigation, and guidance equations are equivalent to those specified in the Launch Vehicle Digital Computer (LVDC) Equation Defining Document for AS-204. The pertinent details of these functions will now be discussed.

A. <u>Missile Dynamics</u>

In simulating the missile dynamics, the accelerations due to thrust, drag, lift, and gravity were considered.



$$T = T_{s\ell} + A_t(p_o - p)$$

where

T = thrust

 $T_{s,\ell}$ = the constant sea level thrust

 A_{+} = thrust chamber area

p = atmospheric pressure

p_o = sea level pressure

Thrust buildup and decay were simulated by an equivalent lengthening of the burn at full thrust.

The thrust levels and mass flow rates were chosen by the following procedure with the intention of matching position and velocity along the reference trajectory:

- 1. The event times shown in Table 1 were selected to reflect the effect of thrust buildups and decays.
- Constant mass flow rates were computed such that the vehicle weight at the events matched the reference.

- 3. $T_{s\ell}$ and A_t were adjusted such that the velocity magnitude and inertial flight path angle matched the reference at the time of jettisoning the launch escape system.
- 4. S-IVB thrust at 5.0:1 mixture ratio was assumed nominal, at 5.5:1 mixture ratio, an average thrust from the reference was used, and at 4.7:1 mixture ratio, the thrust was adjusted to achieve cutoff velocity at the proper time.

Drag and lift forces were computed using the drag and lift coefficient curves given in reference 1 and the Patrick Reference Atmosphere. In computing the normal force, the torque created by the lift force is balanced by deflecting the thrust vector from the roll axis. The gravitational force is computed from the standard Fisher Ellipsoid model of the Earth as defined in reference 7.

The attitude of the vehicle is determined by integrating constant attitude rates. These rates are recomputed each guidance cycle and ordered to accomplish the required change in attitude as determined by the guidance equations. Although angular dynamics and autopilot response are not simulated, rate limiting of 1 degree/second is applied.



B. Inertial Platform Simulation

The accelerometer measurements are simulated by integrating the actual sensed acceleration due to thrust and aerodynamic forces, and adding errors associated with the platform gyros and accelerometers. The error sources considered include the following:

- (1) initial platform misalignment
- (2) gyro drift
- (3) gyro mass unbalance
- (4) gyro anisoelastic effects
- (5) accelerometer misalignment
- (6) accelerometer bias
- (7) accelerometer scale factor error
- (8) accelerometer measurement quantum size

The inertial platform alignment and the orientation of the gyro axes are illustrated in Figure 2. As shown in Figure 3, the operations performed to simulate this platform proceed by first forming the increment in velocity gained during the last guidance cycle due to thrust and aerodynamic forces. This velocity increment $(\overline{\Delta V}_a)$ is expressed in the platform coordinate system which is defined by the nominal platform alignment at guidance reference release.



The effects of initial misalignment and platform drift are simulated by the A matrix. This matrix relates the actual instantaneous platform orientation to the nominal alignment. The initial misalignment is described by three Euler angles taken about the pitch, yaw, and roll axes. The drift rate about each platform axis is given by

$$\omega_{j} = GD_{j} + U_{sj}a_{ij} + U_{ij}a_{sj} + U_{oj}a_{oj} + S_{j}a_{ij}a_{sj}$$

where

 GD_{j} = the fixed gyro drift rate of the jth axis gyro

 $U_{sj}, U_{ij}, U_{oj} =$ the mass unbalance about the spin, input, and output axes of the j^{th} axis gyro

 S_j = the anisoelastic constant of the j^{th} axis gyro, and

 a_{ij}, a_{sj}, a_{oj} = the sensed acceleration along the input, spin, and output axes of the j^{th} axis gyro.

The total drift rate is calculated and used to update the A matrix each computation cycle.

With reference to Figure 3, the B matrix simulates accelerometer misalignment. Since this matrix is not time dependent, it need only be calculated once and is given by

$$[B] = \begin{bmatrix} \cos \varepsilon_{yx} \cos \varepsilon_{xz} & -\sin \varepsilon_{xy} \cos \varepsilon_{yz} & \sin \varepsilon_{xz} \cos \varepsilon_{xy} \\ \sin \varepsilon_{yx} \cos \varepsilon_{yz} & \cos \varepsilon_{yx} \cos \varepsilon_{yz} & -\sin \varepsilon_{yz} \cos \varepsilon_{yx} \\ -\sin \varepsilon_{zx} \cos \varepsilon_{zy} & \sin \varepsilon_{zy} \cos \varepsilon_{zx} & \cos \varepsilon_{zx} \cos \varepsilon_{zy} \end{bmatrix}$$

where the epsilons are the angles by which the accelerometer axes deviate from the platform axes as illustrated in Figure 4. Accelerometer bias and scale factor error are introduced by the vector $\overline{a_b}$ and the matrix [1 + SF] respectively. The scale factor matrix is of the form

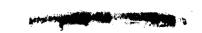
$$\begin{bmatrix}
(1 + SF_{x}) & 0 & 0 \\
0 & (1 + SF_{y}) & 0 \\
0 & 0 & (1 + SF_{z})
\end{bmatrix}$$

where ${\rm SF}_{\rm i}$ represents the scale factor error of the accelerometer along the i $^{\rm th}$ axis.

 $\overline{\Delta V}_{m}$, the output of the platform simulation, now represents the change in the accelerometer measurements that has occurred during the last computation cycle. This quantity is used by the navigation equations, the M/F filter, and the mixture ratio shift sensor.

C. Accelerometer Processing

The accelerometer processing program simulates two significant functions: the mixture ratio shift sensor and the M/F filter. In Section VI.B, the M/F filter specified





in reference 8 is shown to provide an unbiased estimate of the mass intercept time $(\hat{\tau})$ given by

$$\hat{\tau} = (M/F)^S V_{ex}$$

where $(M/F)^S$ is the output of the M/F filter and $V_{\rm ex}$ is the effective exhaust velocity. The mass intercept time is used by the guidance equations to predict the vehicle's acceleration.

D. Navigation Equations

The onboard navigation was simulated by using the accelerometer measurements in the MIT "Average G" equations. These equations were used because they are simpler in form and are demonstrably equivalent to the "Boost Navigation" equations in the LVDC Equation Defining Document. The gravity model used was taken directly from the "Boost Navigation" section of the LVDC document.

E. Guidance Equations

There are two phases of guidance for the AS-204 launch vehicle. During the atmospheric portion of flight, the vehicle attitude is commanded to follow a specified pitch program. This program commands a vertical attitude for 10 seconds, at which time the attitude is ordered to follow a sixth order pitch polynomial. At 133 seconds, the

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attitude is again held constant until active guidance takes command at 185.4 seconds. The coefficients of the sixth order polynomial were calculated by a least squares fit to the gravity turn portion of the open loop reference trajectory which was generated with BCMASP.

After 185.4 seconds, the vehicle is guided by the 2-stage IGM equations described in the LVDC Equation Defining Document. IGM operates in its normal mode until time to go reaches 46 seconds. At this time, the $\widetilde{\chi}$ mode is entered and thereafter IGM controls only the velocity vector at cutoff. When time to go reaches 14 seconds, the last steering orders computed by the IGM equations are flown without updating and a parabolic extrapolation for cutoff time based on velocity is employed.



IV. Vehicle Data

Tables 2 and 3 present the vehicle data used in the error analysis trajectory simulation. Table 2 lists the engine performance data as discussed in section III - A. The weight summary, presented in Table 3, is derived from the Operational Flight Trajectory.



TABLE 2

Engine Performance Data

S-IB Engine Performance Data

1710486. lbs Sea level thrust 6226.655 lbs/sec Weight rate 6638.749 in² Thrust chamber area 360. ft² Drag area

S-IVB Engine Performance Data

5.0:1 Mixture Ratio

200000. lbs Thrust 504.724 lbs/sec Weight rate 5.5:1 Mixture Ratio

226200. lbs Thrust 534.6 lbs/sec Weight rate

4.7:1 Mixture Ratio

Weight rate

190220. lbs Thrust 441.728 lbs/sec



TABLE 3

Weight Summary

Inert wgt at cutoff	65,070	
Fuel reserve at cutoff	4,062	
Injection weight		69,132
S-IVB weight consumed	224,681	
Launch escape system	8,500	
Ullage cases	220	
S-IVB weight at separation		302,533
S-IB weight consumed	891,190	
S-IB inert weight	102,645	
Vehicle liftoff weight		1,296,368

V. Error Analysis Results

The error analysis results corresponding to three-sigma values of platform and performance errors are compiled in this section. The platform uncertainties were derived from reference 10, and the performance uncertainties were obtained from reference 11.

A. Errors at S-IVB Cutoff

Tables 4 through 7 present the errors at S-IVB cutoff that were generated by simulating perturbed closed loop trajectories for each error source. The tabulated differences represent the actual state on the perturbed trajectory minus the actual state on the nominal. Definitions of all symbols are listed in the Appendix.

In computing the root-sum-square (RSS) values, the larger of the differences obtained from the positive and negative perturbation of each error source was used. The RSS values represent the three-sigma magnitudes of variations in the parameters at cutoff.

Total three-sigma values, considering both platform and performance uncertainties, are listed below for the significant parameters at S-IVB cutoff.

Radius 1492.6 ft

Velocity magnitude 3.981 ft/sec

Flight path angle .0131 deg



Inclination

.0068 deg

Node angle

.0252 deg

Time from launch

19.217 sec

S-IVB fuel reserve

1579.5 lbs

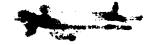
The velocity magnitude uncertainty shown above also includes the effects of quantization on the cutoff time computation (section VI D) and a .03 second three-sigma uncertainty in the cutoff execution. The cutoff execution uncertainty contributes 2.61 ft/sec as computed from

$\Delta V \approx a_{T} \cos \alpha \Delta t$

where a_T is the axial acceleration (88.6 ft/sec²) and α is the angle of attack (11 deg) at S-IVB cutoff.

Figures 5 and 6 show S-IVB fuel reserve as a function of S-IB and S-IVB fuel loading respectively. The figures indicate that the fuel reserve could be increased by loading additional fuel in both stages.

Fuel optimization for the zero lift trajectory was investigated. In Section II, it was noted that the gravity turn was selected to match the altitude of the MSFC trajectory at 185.40 seconds. Figure 7 demonstrates that the S-IVB cutoff time could be decreased by .5 seconds by reshaping the trajectory. Since this results in a fuel saving of only



220 lbs, the MSFC reference trajectory is a near optimum zero lift trajectory. (Lifting trajectories may add to the fuel reserve, 13 but were not considered in this report.)

The uncertainty in fuel reserve is controlled by variations in first and second stage $I_{\rm sp}$ (no mass rate change). For the assumed three-sigma $I_{\rm sp}$ variation of 1%, the probability of fuel depletion is extremely small. Figures 12 and 13 are graphs showing the variation of the three-sigma fuel reserve and the probability of fuel depletion with changes in the assumed value of $I_{\rm sp}$ uncertainties and nominal fuel reserve.

B. Errors at 640 Seconds

Tables 8 through 11 present the errors in local vertical coordinates at 640 seconds, approximately 50 seconds after nominal S-IVB cutoff. This coordinate system and all symbols are defined in the Appendix. The values tabulated in Tables 8 and 9 represent differences in the actual state (perturbed-nominal), whereas those tabulated in Tables 10 and 11 represent differences in the estimated state (perturbed-nominal) as determined by the onboard launch vehicle navigation equations.

The errors are presented in this form to provide data for generating transition matrices between fixed times on the AS-204 trajectory.



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VI. Analytical Investigations

The purpose of this section is to determine analytically the performance of certain system functions which
operate in the presence of noise. The specific areas studied
are:

- (1) Noise and dynamic response of the M/F filter.
- (2) Noise response of the Navigation equations (Average G Equations).
- (3) The effect of noise on the prediction of the cutoff time.

The noise source considered is the quantization in the accelerometer registers. A model for this noise source is developed.

A. Noise Model

At a set of discrete time instants, $\{t_j\}$ $j=0,1,\ldots,n$, velocity measurements $V_i(t_j)=1,2,3$ are made by three integrating accelerometers mounted orthogonally on an inertial platform

$$\overline{V}_{a}(t_{j}) = \int_{0}^{t_{j}} a_{T}(t) \overline{I}(t) dt + \overline{\delta}(t_{j})$$
 (1)

Here

(a)
$$\overline{V}_{a}(t_{j}) = \begin{bmatrix} V_{1}(t_{j}) \\ V_{2}(t_{j}) \\ V_{3}(t_{j}) \end{bmatrix}$$
 (2)

represents the vector of accelerometer readings.

- (b) $a_{T}(t)$ represents the magnitude of the thrust acceleration.
- (c) $\overline{\mathbf{\ell}}(t)$ is the direction cosine vector of the thrust acceleration vector in platform coordinates.
- (d) $\overline{\delta}(t)$ represents the vector of errors introduced by quantization.

For the purpose of this analysis it will be assumed that the thrust acceleration is such as to cause the velocities in all three platform coordinates to be monotonically increasing functions of time. With this assumption together with zero initial velocity stored in the accelerometers, the accelerometer readings will be lower than the true values. Each component of the quantization noise thus has the probability density function

$$p(\delta_{i}(t_{j})) = \begin{cases} \frac{1}{b} & -b < \delta_{i}(t_{j}) < 0 \\ 0 & \text{elsewhere} \end{cases}$$
 (3)



where b = .164 ft/sec (.05 meters/sec) is the value of the least significant bit in each accelerometer register.

Thus the mean of the quantization noise is

$$E\{\delta_{i}(t)\} = -\frac{b}{2} \text{ ft/sec} = -.082 \text{ ft/sec}$$
 (4)

and the variance about the mean is

$$E\left\{\left(\delta_{i}(t) + \frac{b}{2}\right)^{2}\right\} = \frac{b^{2}}{12} \text{ ft}^{2}/\text{sec}^{2} = .00224 \text{ ft}^{2}/\text{sec}^{2}$$
 (5)

The noise is also assumed to be uncorrelated, i.e.,

$$E\left\{\left(\delta_{\mathbf{j}}(t_{\mathbf{j}}) + \frac{b}{2}\right)\left(\delta_{\mathbf{j}}(t_{\mathbf{k}}) + \frac{b}{2}\right)\right\} = 0 \quad \text{for } \mathbf{j} \neq \mathbf{k}.$$

The above model has assumed zero initial conditions for the accelerometers. However, since the accelerometers are running before launch, the initial conditions are actually random variables which are uniformly distributed over the quantum interval. The effects of these initial conditions are included in the analysis.

B. Noise and Dynamic Response of the M/F Filter

The change in velocity over a one cycle interval
can be obtained from (1).



$$\overline{V}_{a}(t_{j}) - \overline{V}_{a}(t_{j-1}) = \Delta \overline{V}_{a}(t_{j}) = \int_{t_{j-1}}^{t_{j}} a_{T}(t) \overline{I}(t) dt + \overline{\delta}(t_{j}) - \overline{\delta}(t_{j-1})$$

$$\Delta \overline{V}_{a}(t_{j}) = \Delta \overline{V}(t_{j}) + \Delta \overline{\delta}(t_{j})$$
 (6)

Here $\Delta \overline{V}_a$ represents the measured change in velocity and $\Delta \overline{V}(t_i)$ represents the actual change in velocity.

A raw estimate (F/M) of thrust acceleration can be formed by

$$(F/M)_{j} = \frac{\parallel \Delta \overline{V}_{a}(t_{j}) \parallel}{\Delta T}$$
 (7)

where $\| \overline{x} \|$ represents the Euclidean norm $\left(\sum_{i=1}^{3} x_i^2 \right)^{1/2}$

of the enclosed three dimensional vector and $\Delta T = t_j - t_{j-1}$ is nominally 1.7 seconds.

In the absence of noise F/M closely approximates the true value of thrust acceleration which is inversely proportional to a linear function of time.

In order to obtain an estimate of a_T , a filter has been designed which operates on the (M/), data points, which nominally lie on a straight line $\left(M/F \approx \frac{1}{a_T}\right)$ which is linear.

The filter specification is obtained by digitalizing the following transfer function



$$\frac{(M/F)^{s}}{(M/F)} = \frac{1 + 42s}{1 + 42s + 520s^{2} + 1848s^{3} + 1936s^{4}}$$
(3)

When this transfer function is put into the form of a negative feedback system, the resulting system is seen to be of type 2. Thus the continuous filter will follow a ramp input (which is the expected input) with no steady state error.

The transfer function given by (8) is digitalized to yield the following difference equation:

$$(M/F)_{i}^{s} = 2.8048253(M/F)_{i-1}^{s} -2.8588037(M/F)_{i-2}^{s}$$

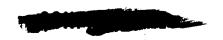
$$+1.2493337(M/F)_{i-3}^{s} -0.19735971(M/F)_{i-4}^{s}$$

$$+0.012095511(M/F)_{i} +0.021333516(M/F)_{i-1}$$

$$-0.026158031(M/F)_{i-2} -0.0052666918(M/F)_{i-3}$$
 (9)

The discrete time filter has a steady state error for a ramp input. This error corresponds to a time lead of .85 seconds which is $\Delta T/2$. If it is assumed that $(F/M)_j$ computed from (7) is valid at $t_{j-1} + \frac{\Delta T}{2}$, the total time delay is zero, thereby providing an unbiased estimate.

In order to obtain the noise response of the filter it is necessary to solve the difference equation (9). Solution of this equation results in the filter weighting sequence which can be used to obtain the output noise variance.





Equation (9) may be transformed into the z domain to yield the following transfer function

$$H(z) = \frac{N(z)}{D(z)}$$

$$N(z) = 0.012095511z^{4} + 0.021333516z^{3}$$

$$-0.026158031z^{2} - 0.0052666918z$$

$$D(z) = (z-z_{1})(z-z_{2})(z-z_{3})(z-z_{4})$$

where

$$z_1 = 0.55836431$$
 $z_2 = 0.89341121$
 $z_3 = 0.92563474$
 $z_4 = 0.42741504$

 $\label{eq:continuous} \text{Inverting $H(z)$ yields the weighting sequence valid} \\$ for k>0

$$h(k) = -0.3851741(z_1)^{k-1} + 0.5297386(z_2)^{k-1}$$

$$-0.25246(z_3)^{k-1} + 0.1631549(z_4)^{k-1}$$
(10)

and

$$h(0) = 0.012095511$$



Figure 8 shows a plot of this function and Figure 9 shows the unit step response which is given by

$$(M/F)_n^s = \sum_{i=0}^n h(i)$$

For an input which consists of uncorrelated, zero mean, stationary noise samples of variance $\sigma_{\rm in}^2$, the standard deviation of the output noise is given by

$$\sigma_{\text{out}}(k) = \left[\sum_{i=0}^{k} h^{2}(i)\right]^{1/2} \sigma_{\text{in}}$$

It is seen from Figure 10 that $\sigma_{\mbox{out}}$ settles to .323 $\sigma_{\mbox{in}}$ resulting in a 67.7% noise reduction.

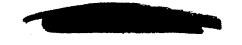
C. Noise Response of the Powered Flight Navigation Equations ("Average G Equations")

It has been shown 9 that the Saturn navigation equations can be expressed by

$$\overline{R}_{n} = \overline{R}_{n-1} + \overline{V}_{n-1}\Delta T + \frac{1}{2} \left[\Delta \overline{V}_{a}(n) + \overline{G}_{n-1}\Delta T\right]\Delta T \tag{11a}$$

$$\overline{G}_n = \overline{G}(\overline{R}_n)$$

$$\overline{V}_{n} = \overline{V}_{n-1} + \Delta \overline{V}_{a}(n) + \frac{1}{2} \left[\overline{G}_{n} + \overline{G}_{n-1} \right] \Delta T$$
 (11b)



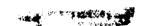
where

- a) $\Delta \overline{V}_a$ is the change in velocity, due to thrust acceleration, as measured by the accelerometers.
- b) $\overline{\mathbb{R}}_n$ is the estimate of the position vector at time \mathbf{t}_n .
- c) $\overline{\mathbb{V}}_n$ is the estimate of the velocity vector at time \mathbf{t}_n .
- d) \overline{G}_n is the gravitational acceleration vector which is a non-linear function of \overline{R}_n .
- e) AT is the computation cycle time.

In a recent paper 15 the effect of noise in ΔV_a on the output of these equations was analytically determined. The one dimensional problem was considered and the model used for gravity was

$$G_{n} = - \mu/R_{n}^{2}$$
 (llc)

Equations (11) represent a set of non-linear, coupled, difference equations for the navigation position and velocity (the non-linearity being due to the gravity term). In order to determine the statistics of the errors resulting from noisy measurements (ΔV_a) the gravity equation





(llc) was linearized about a nominal value. The position equation (lla) was then uncoupled from (llb) in such a way as to yield a second order difference equation for the noise in position. The solution of this equation yielded a weighting sequence (impulse response) which was used to determine the statistics of the resulting position errors. The weighting sequence was then used to determine the statistics of the navigation velocity errors introduced by the coupling through the gravity terms.

In this section the results derived in Reference 15 will be applied to an error analysis of the Saturn launch vehicle navigation equations. The error source considered here is quantization in the accelerometer registers. A model for quantization has been formed in Reference 14 and the resulting statistics are

$$E\{\delta V_a\} = -.082$$
 ft./sec.

and

$$E\{(\delta V_a + .082)^2\} = \sigma_a = 0.0474$$
 ft./sec.

The results quoted here will be for n = 353 and ΔT = 1.7 seconds (n ΔT \approx 600 seconds).

By using the equations derived in Reference 15 including the effects of initial conditions, the mean of the position error is found to be

$$E\{\delta R_{353}\} = 0. ft.$$

and the standard deviation about this mean is

$$\sigma_{\delta R} = 34. \text{ ft.}$$

The mean of the velocity error is found to be

$$E\{\delta V_{353}\} = 0. \text{ ft./sec.}$$

and the standard deviation about this mean is

$$\sigma_{\delta V_{353}} = 0.047 \ (\sqrt{2}) = 0.066 \ \text{ft./sec.}$$

Here the standard deviation of the velocity error is due to the noise in the most recent accelerometer measurement and the random initial conditions.

The conclusion of this analysis is that the output of the position navigation equation is unbiased and has a standard deviation of 34. feet. The output of the velocity navigation equation is also unbiased and has a standard deviation of 0.066 ft./sec.





D. The Effect of Noise on the Prediction of the Cutoff Time

In this section the scheme used to determine the $\operatorname{\mathtt{cutoff}}$ time $\operatorname{\mathtt{^*}}$ of the S-IVB will be evaluated.

The procedure used is to fit a second degree polynomial to the magnitude of the three most recent velocities obtained from the navigation equations. The velocity data is then extrapolated, using the polynomial, to determine the time at which the predicted velocity would equal the desired terminal velocity $(V_{\rm T})$.

Let v_2 , v_1 , v_0 be the three most recent velocities obtained from the navigation equations at times t_2 , t_1 , and t_0 respectively, where

$$t_2 > t_1 > t_0$$

$$t_2 - t_1 = \Delta t_2, \quad t_1 - t_0 = \Delta t_1$$

and

$$t_2 - t_0 = \Delta t_3$$

It is assumed that the scheme used is that outlined in the LVDC Equation Defining Document for the AS-204 Flight Program.5

The resulting extrapolating polynomial is

$$a_2 t_g^2 + a_1 t_g + V_2 - V_T = 0$$
 (12)

where $\mathbf{t}_{\mathbf{g}}$ is the predicted time to go required to achieve $\mathbf{V}_{T}.$ The coefficients of the polynomial are

$$a_2 = \frac{(V_2 - V_1)\Delta t_1 - (V_1 - V_0)\Delta t_2}{\Delta t_2 \Delta t_1 \Delta t_3}$$

$$a_1 = \frac{V_2 - V_1}{\Delta t_2} + a_2 \Delta t_2$$

The method proposed to solve (12) for \boldsymbol{t}_g is recursive

$$t_g = \frac{V_T - V_2}{a_1 + a_2 t_g}$$

where the t_g appearing in the denominator is the previous estimate minus Δt_2 , i.e.,

$$(t_g)_{t_2} = \frac{v_T - v_2}{a_1 + a_2 \left[(t_g)_{t_1} - \Delta t_2 \right] }$$
 (13)



Since a_2 is small the quadratic is not solved directly. For the purpose of the noise analysis of the cut-off scheme it will be assumed that (13) is an exact solution of (12).

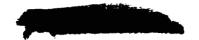
Let the velocity output of the navigation equations be represented by

$$V(t) = P(t) + n(t) + \delta V_{O}$$
 (14)

where

- (a) P(t) represents the actual velocity assumed to be a second degree polynomial in time.
- (b) n(t) is the noise output of the navigation equations due to quantization. The one sigma value of n(t) is .0474 ft/sec. It should also be remembered that n(t) is an uncorrelated process.
- (c) δV_{O} represents the initial conditions of the accelerometers at liftoff. The one sigma value of δV_{O} is also .0474 ft/sec.

Since δV_0 is a constant during the flight, it affects all computations of V and thus affects the terminal velocity directly. However, since n(t) is an uncorrelated process, the standard deviation of the error in terminal velocity (σ_0) due to n(t) is a function of the number of prediction cycles and the level of the input noise $(\sigma_i(t))$ as shown in Figure 11.



For the problem under consideration,

$$\sigma_i = .0474 \text{ ft/sec.}$$

and M, which is the number of points used in the fit, is 3. The cutoff calculation is terminated when the prediction time is between 0.8 and 1.8 cycles, and the current estimate of $t_{\rm g}$ is used to command cutoff.

The prediction time (t_p) is now treated as a random variable uniformly distributed from 0.8 cycles to 1.8 cycles. The density function for t_p is

$$p(t_p) = \begin{cases} \frac{1}{\Delta T} & 0.8\Delta T < t_p < 1.8\Delta T \\ 0 & \text{elsewhere} \end{cases}$$

It can be shown that the conditional variance of the velocity error at cutoff (\mathbf{t}_{co}) given the prediction time is given by

$$E\left\{\left[P(t_{co}) - V_{T}\right]^{2}/t_{p}\right\}$$

$$= \sigma_{i}^{2}\left[1 + \frac{3t_{p}}{\Delta T} + \frac{15}{2}\left(\frac{t_{p}}{\Delta T}\right)^{2} + 6\left(\frac{t_{p}}{\Delta T}\right)^{3} + \frac{3}{2}\left(\frac{t_{p}}{\Delta T}\right)^{4}\right]$$

Now the variance of the velocity error at cutoff is given by

$$\sigma_0^2 = E\left\{ \left[P(t_{co}) - V_T \right]^2 \right\} = \int_{-\infty}^{\infty} E\left\{ \left[P(t_{co}) - V_T \right]^2 / t_p \right\} p(t_p) dt_p.$$



Letting $x = \frac{t_p}{\Delta T}$ yields

$$\frac{\sigma_0^2}{\sigma_1^2} = \int_{0.8}^{1.8} \left(1 + 3x + \frac{15}{2}x^2 + 6x^3 + \frac{3}{2}x^4\right) dx = 38.9$$

or

$$\frac{\sigma_0}{\sigma_i} = 6.23.$$

Thus the standard deviation of the error in cutoff velocity due to quantization in the measurements during the flight and initial velocity stored in the accelerometers at liftoff is

$$\sigma = (.0474) \left\{ (6.23)^2 + (1)^2 \right\}^{\frac{1}{2}} = .299 \text{ ft/sec.}$$

The three sigma deviation of .897 ft/sec. has been included in the velocity magnitude uncertainty listed on page 17.

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APPENDIX

Definition of Coordinate Systems

Platform Coordinates

The platform coordinate system (x,y,z) is an earth centered inertial system defined with the y-axis upward along the local vertical at launch, the x-axis perpendicular to the y-axis, pointing downrange along the launch azimuth, and the z-axis completing a right handed orthogonal coordinate system.

UVW Coordinates

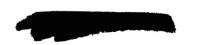
The UVW coordinate system is an earth centered inertial system defined relative to the state vector at 640 sec on the nominal closed loop reference trajectory. U is defined as upward along the position vector, V is along the projection of the velocity vector in a plane perpendicular to U, and W completes the right handed orthogonal system.

DEFINITION OF SYMBOLS

Observed Trajectory Variables

Symbol	Units	Description
XDOT	ft/sec	(
YDOT	ft/sec	velocity in platform coordinates
ZDOT	ft/sec	(

Symbol	Units	Description
Х	ft	1
Y	ft	position in platform coordinates
Z	ft	
V	ft/sec	magnitude of velocity vector
R	ft	magnitude of position vector
Т	sec	time from launch
BETAI	deg	inertial flight path angle
WGT	1bm	mass of vehicle
PHIT	deg	central angle flown by vehicle
GINCL	deg	inclination of orbit
DNODE	deg	angle from launch meridian to descending node
VU	ft/sec	
vv	ft/sec	velocity in UVW coordinates
VW	ft/sec	
RU	ft	
RV	ft	position in UVW coordinates
RW	ft	
VN U	ft/sec	I marriantian actimate of valenity
VNV	ft/sec	navigation estimate of velocity
VNW	ft/sec	in UVW coordinates
RNU	ft	novigation againsts of magitism
RNV	ft	navigation estimate of position
RNW	ft	in UVW coordinates





Platform Errors

Symbol	Units	Description
GDSVX	deg/hr	latardy state some design mater
GDSVY	deg/hr	steady state gyro drift rates
GDSVZ	deg/hr	about X, Y, and Z axes
USSVX	deg/hr/g	
USSVY	deg/hr/g	gyro drift due to mass unbalance
USSVZ	deg/hr/g	about spin axis
UISVX	deg/hr/g	
UISVY	deg/hr/g	gyro drift due to mass unbalance
UISVZ	deg/hr/g	about input axis
UOSVX	deg/hr/g	1
UOS VY	deg/hr/g	gyro drift due to mass unbalance
UOSVZ	deg/hr/g	about output axis
SSVX	deg/hr/g ²	
SSVY	deg/hr/g ²	gyro drift due to anisoelastic
SSVZ	deg/hr/g ²	effects .
SFSVX	-	
SFSVY	-	accelerometer scale factors
SFSVZ	-	
ABSVX	g	
ABSVY	g	accelerometer bias
ABSVZ	g	



Symbol	Units	Description
CHILX	deg	1
CHILY	deg	initial platform misalignment
CHI1Z	deg	
EPSVXY	deg	
EPSVXZ	deg	
EPSVYX	deg	accelerometer misalignment co-
EPSVYZ	deg	efficients (see Figure 4)
EPSVZX	deg	
EPSVZY	deg	

Performance Errors

Symbol	<u>Units</u>	Description
HEDWND	ft/sec	horizontal wind in pitch plane,
		opposing vehicle's motion
RCRWND	ft/sec	horizontal wind from right side
		along pitch axis
EISP1	%	error in first stage specific im-
		pulse, weight rate held constant
EDWGT1	%	error in first stage weight rate,
		specific impulse held constant
EISP2	%	error in second stage specific im-
		pulse, weight rate held constant

Appendix - 5

Symbol	<u>Units</u>	Description
EDWGT2	%	error in second stage weight rate,
		specific impulse held constant
WGTl	lbm	error in first stage dry mass
EFUELl	%	error in first stage fuel loading
EFUEL2	%	error in second stage fuel
		loading
ETMIX2.	sec	error in time of mixture ratio
		shift
ECD	%	error in drag coefficient
ECL	%	error in lift coefficient
ERHOA	%	error in atmosphere's density

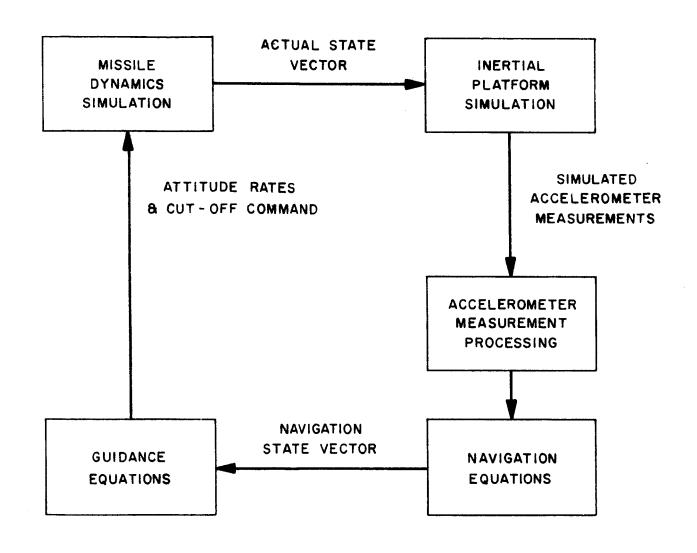


FIGURE 1
CLOSED LOOP SIMULATION

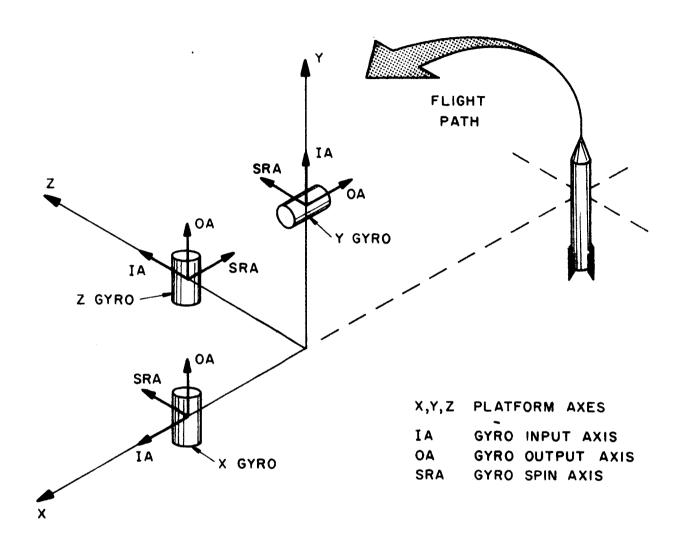


FIGURE 2
PLATFORM CONFIGURATION

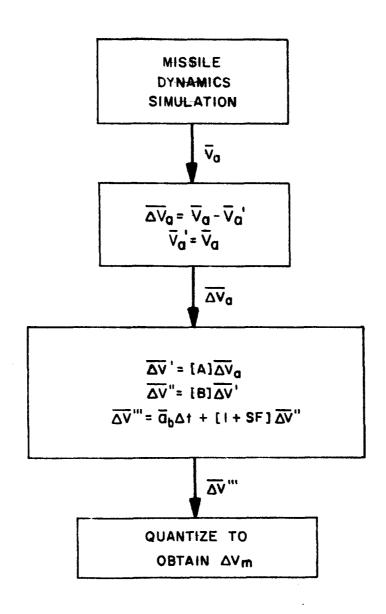


FIGURE 3
INERTIAL PLATFORM SIMULATION

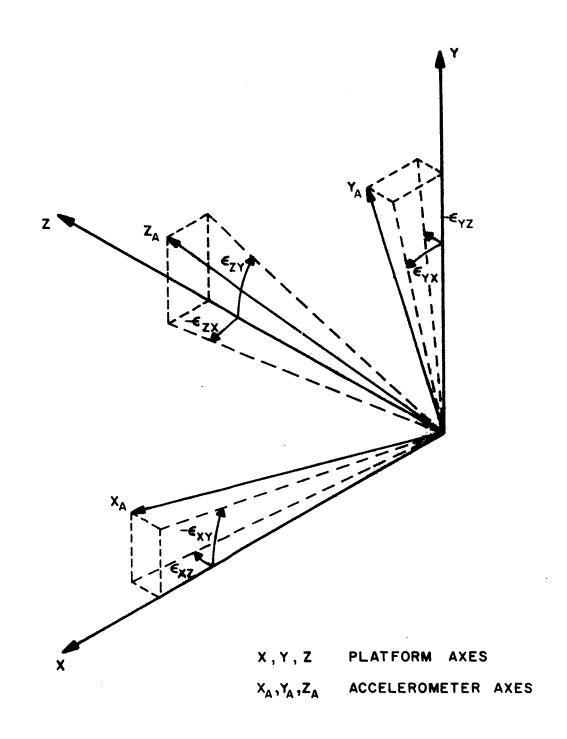
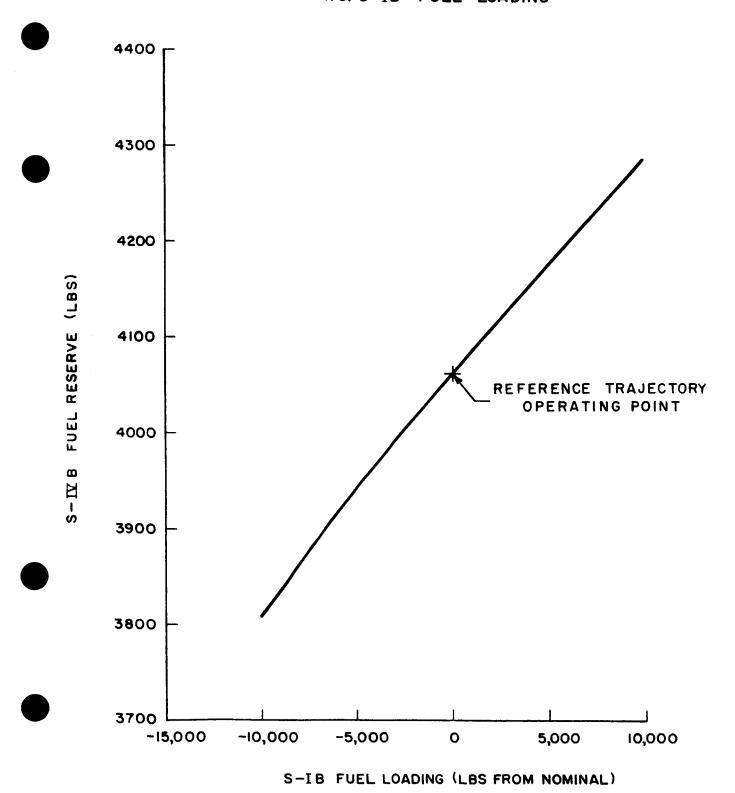


FIGURE 4
ACCELEROMETER MISALIGNMENT

FIGURE 5
S-IVB FUEL RESERVE
V. S. S-IB FUEL LOADING



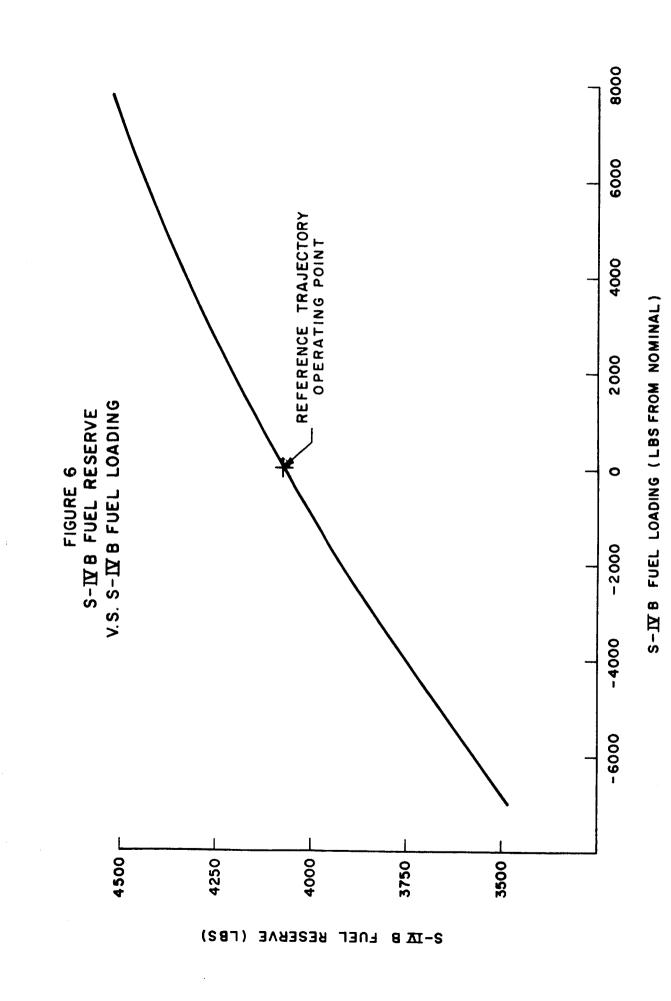
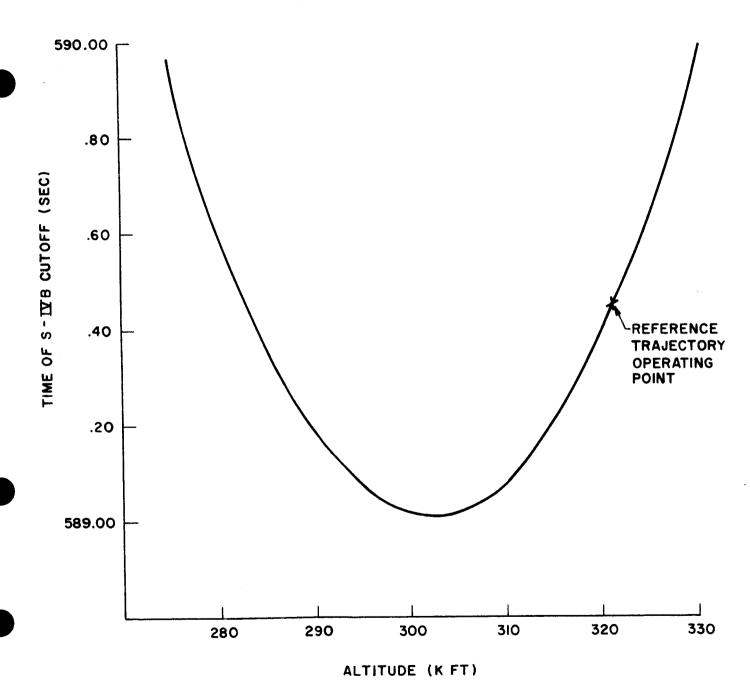
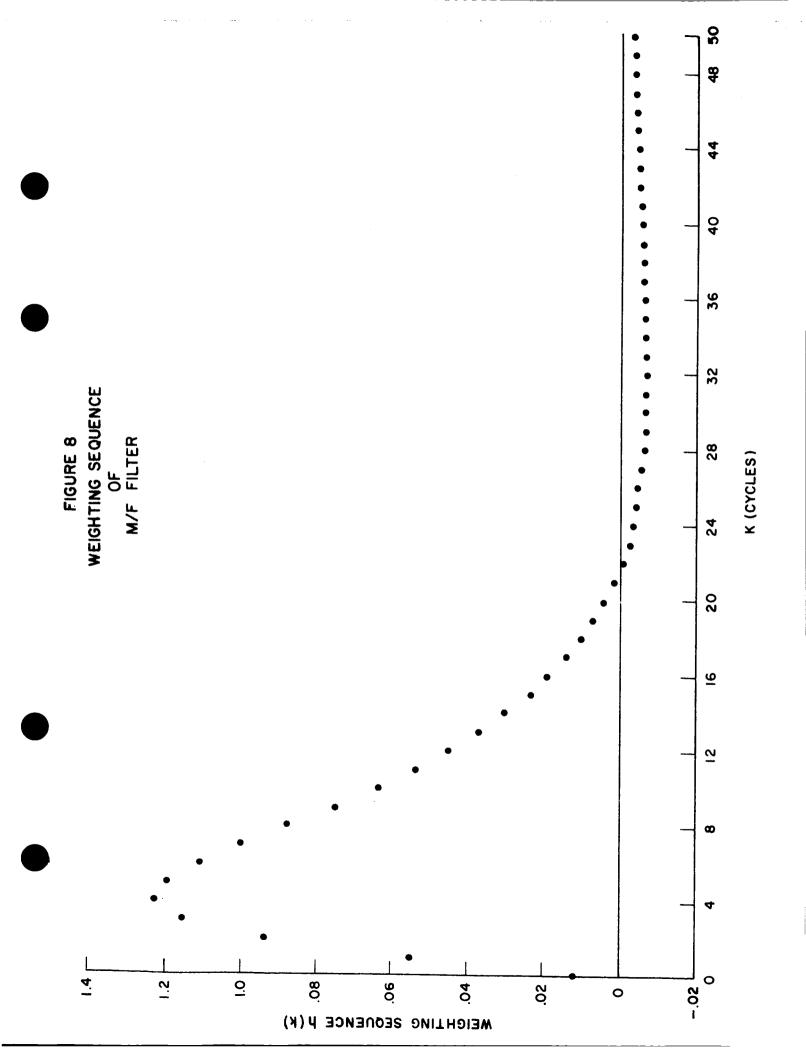


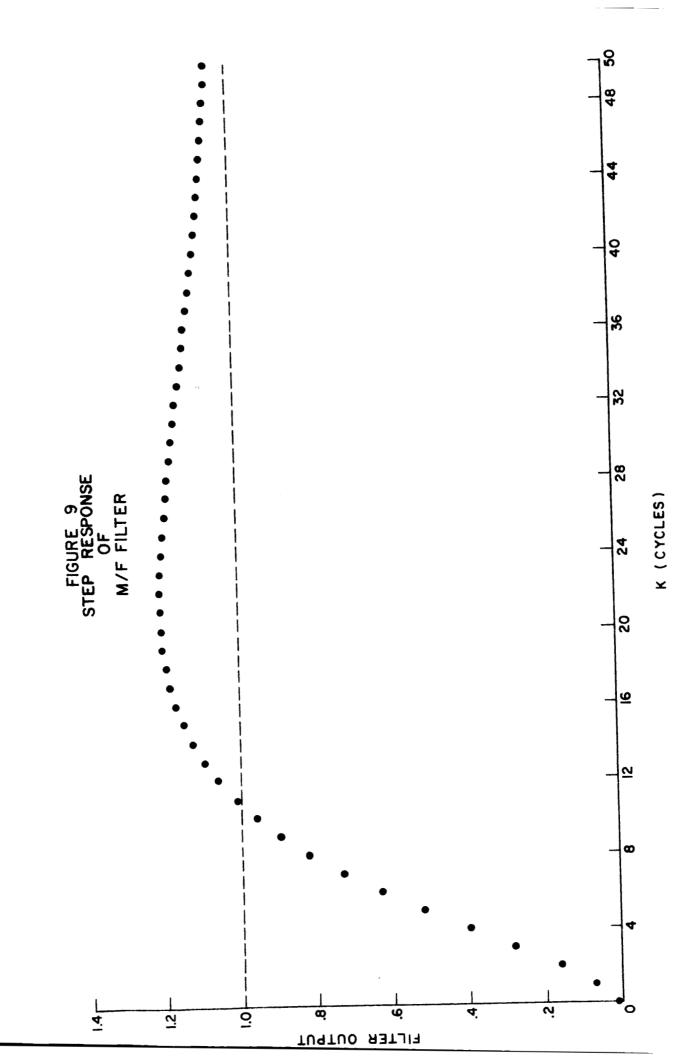
FIGURE 7

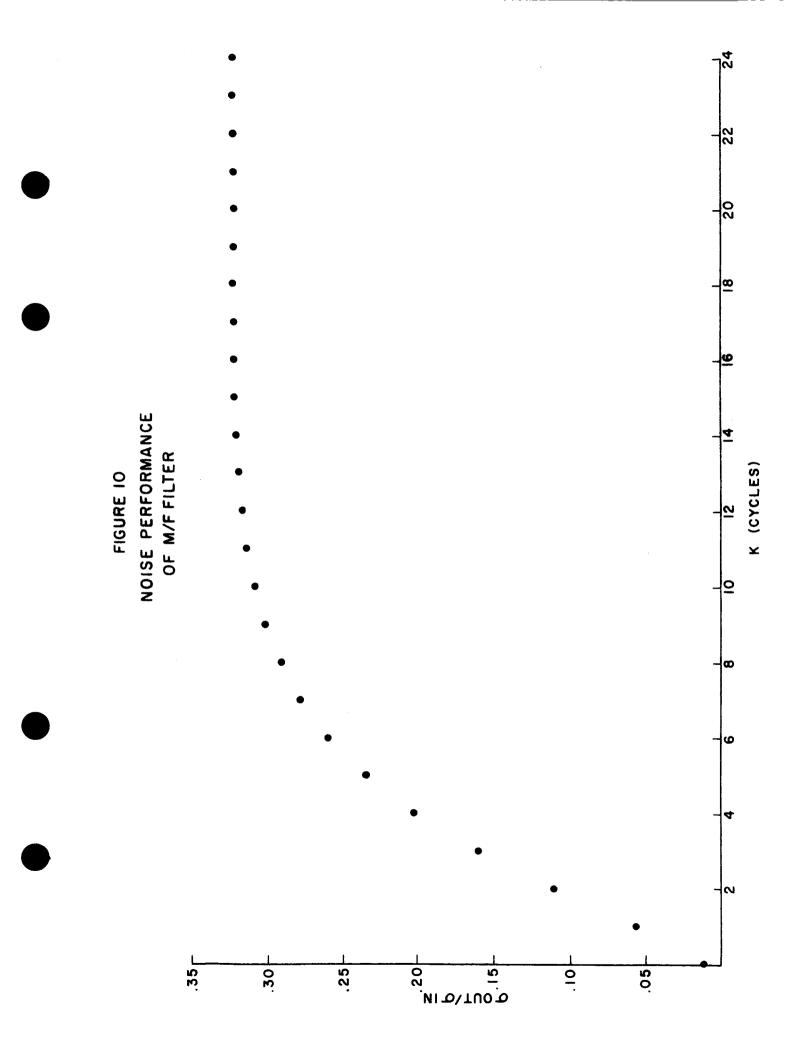
TIME OF S-IVB CUTOFF VS

ALTITUDE AT 185.4 SECONDS









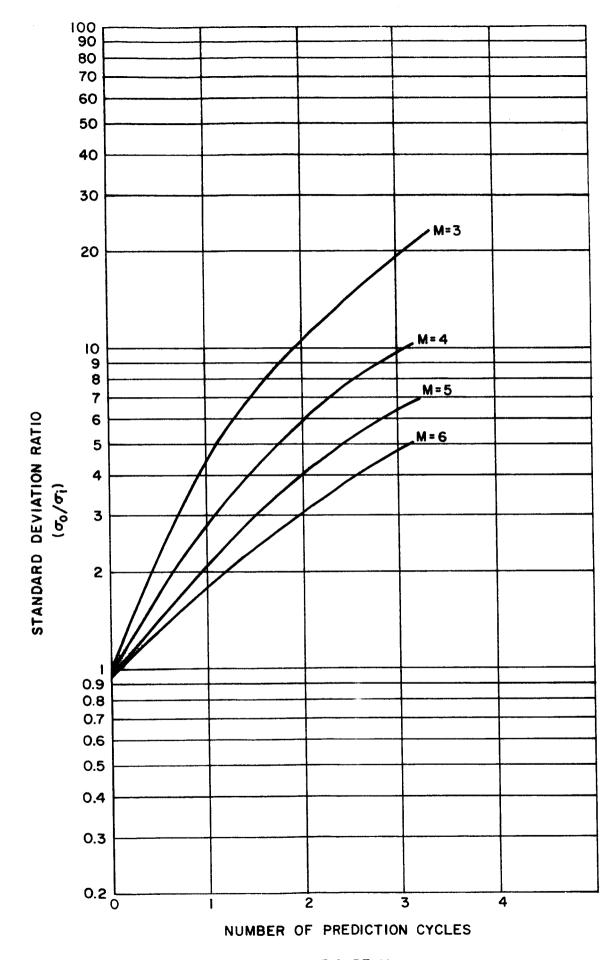
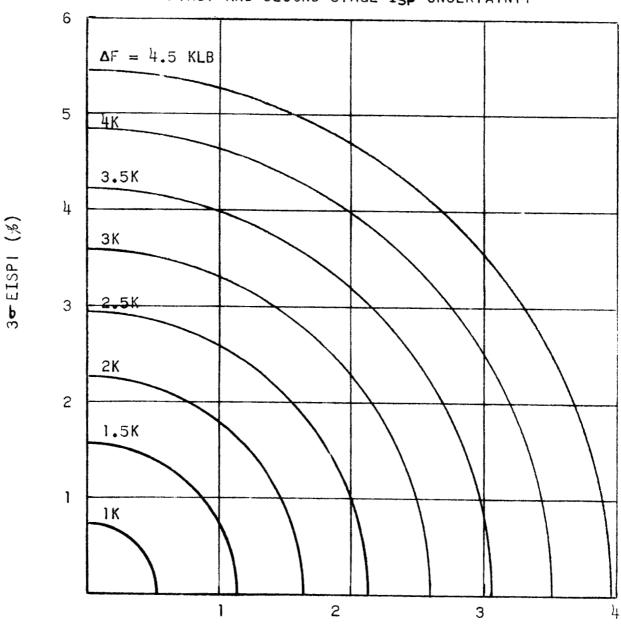


FIGURE 11

FIGURE 12 $$3\sigma$$ FUEL UNCERTAINTY AS A FUNCTION OF FIRST AND SECOND STAGE $I_{\mbox{\footnotesize{\bf SP}}}$ UNCERTAINTY



3 **~** EISP2 (%)

FIGURE 13 PROBABILITY OF FUEL DEPLETION AS A FUNCTION OF FUEL RESERVE AND FUEL UNCERTAINTY FUEL RESERVE (K LB) **∽** △F (K LB)